Diffusive and Fast Filter Using Ierative Gaussian Smoothing

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ABSTRACT

A fast version of the iterative Gaussian smoothing, which can properly remove the non-sinusoidal part of a data string with an operation count in the same order of applying a Fast Fourier Transform (FFT), say less than $2N\ln(N) + 100N$, is proposed. In order to shrink the transition zone of the Gaussian smoothing method, in a series of recent studies, the iterative Gaussian smoothing method had been constructed by repeatedly smoothing the high frequency part. Like the Gaussian smoothing, the resulting smooth part had been proven to be approximately diffusive with respect to the original data string too. However, this promising procedure requires a long computing time. In order to speed up the turn around time, the estimated attenuation factor of the iterative scheme, which is in a close form, is employed to construct an approximate and fast iterative method. The spectrum of the remaining high frequency is obtained immediately after multiplying every Fourier mode by the corresponding attenuation factor with proper power. In this study, the linear trend removal is employed to reduced the error magnitude due to the missing data beyond the interested region. A few tests were examined to show the performance of the proposed procedure.

Keywords: iterative Gaussian smoothing method, spectral method, linear trend removal

1. INTRODUCTION

Because of the development of computer hardware and software, the capability of collecting long data strings is rapidly increasing. Engineering applications of these data heavily rely on the understanding of the involved details. It seems that the most convenient tool to look into the details of a data string is the FFT algorithm [1,2,3]. Currently, most FFT algorithms evaluate the spectrum without any treatment upon the data string. Although the non-sinusoidal part of a data string has a certain contribution to almost every mode, the corresponding spectral studies were successfully carried out in the medium and high frequency zones where this Direct Current (DC) contamination is not serious [1,2,3]. In fact, there are many engineering problems related to the low frequency zone. For example, in a complicated system involving many rotational components, such as bearings, rotators, and wheels etc., the vibration condition induced by the sub-harmonic modes of these rotating parts may seriously relate to the system performance and/or failure. Consequently, detailed studies concerning the low frequency zone were not fully developed in many engineering problems.

To the authors' knowledge, a potential method to remove the non-sinusoidal part is the Gaussian smoothing method [4-6] which was known to be widely applied in many fields after the work of Marr and Hildreth in 1980 [7]. In Ref.[8], properties of smoothing methods were discussed and higher-order differentiation filters were developed. Ray and Ray [9] had made an interesting extension to repeatedly smooth the smooth part.

In order to reduce the DC contamination, ref.[10] restudied the Gaussian smoothing method. The relation between the original data and resulting smoothed data was proven to be the numerical approximation of that observed between the initial temperature distribution and the exact solution at a corresponding instant of the equation for unsteady one-dimensional heat conduction [11]. Since the resulting transition zone of applying the Gaussian smoothing is too wide to service as a filter, an iterative Gaussian smoothing different from that of Ref.[10] was developed.

In later studies [12,13], if the original data involves a polynomial of finite degree, the resulting high frequency part of applying the Gaussian smoothing using a proper smoothing window width σ, was proven to reduce the degree of the polynomial by 2. Suppose that the non-sinusoidal part embedded in a data string can be properly approximated by a polynomial of finite degree, the sinusoidal part can be extracted by the iterative filter. Unfortunately, although the iterative filter effectively shrinks the transition zone, it required too much computing resource. Therefore, the main concerning is to speed up iterative filter.

In this study, the Gaussian smoothing and iterative filter will be briefly restated. Next, the procedure to determine number of iteration cycle and Gaussian window size will be show. Then, the proposed spectral method and error estimation will be presented. Finally, two tests will be examined to show the performance of the new procedure.

2. THEORETICAL DEVELOPMENT

1. Previous Works

1.1 Gaussian Smoothing method

Consider a set of infinite data (t_i, y_i), $t_i = i\Delta t$, $-\infty < i < \infty$, to be smoothed. The zero-th order moving least-squares method weighted by the Gaussian function will give

$$\bar{y}_j = \frac{1}{k}\sum_{i=-\infty}^{\infty} \exp[-\frac{(i-j)^2(\Delta t)^2}{2\sigma^2}]y_i, \ -\infty < j < \infty$$
$$k = \sum_{i=-\infty}^{\infty} \exp[-\frac{(i-j)^2(\Delta t)^2}{2\sigma^2}] \approx \sqrt{2\pi}\sigma/(\Delta t) \quad (1).$$

As $\Delta t \to 0$, $k \to \sqrt{2\pi}\sigma/(\Delta t)$ and the above formula becomes the explicit form shown in Ref.[*-*]. Assume y_j can be expressed in the following form

$$y_j = \sum_{l=0}^{\infty}[c_l \cos\frac{2\pi t_j}{\lambda_l} + d_l \sin\frac{2\pi t_j}{\lambda_l} + \sum_{n=0}^{M} p_n t_j^n] \quad (2),$$

where λ_l is the wavelength of the $l-$th mode and the second summation represents the non-sinusoidal part and M is referred to as the largest power for which $p_n \to 0$ for all $n > M$. For a finite data string with a period of T, $\lambda_l = T/l$, the upper limit of the summation is a finite integer and Eq.(3) is just a Fourier series expansion. Ref.[10,12,13] showed that the following filtering procedure works very well without knowing the exact form of the spectrum, involving a prior knowledge of the values of c_l's, d_l's and λ_l's. After applying the Gaussian smoothing method, it can be shown that the response is [10,12-13]

$$\bar{y}_j = \sum_{l=0}^{\infty} a(\sigma,\lambda_l)\left[c_l \cos\frac{2\pi t_j}{\lambda_l} + d_l \sin\frac{2\pi t_j}{\lambda_l}\right] + \sum_{n=0}^{M} p_n t_j^n + \sum_{n=0}^{M-2} q_{n,1} t_j^n + O(\Delta t^2) \quad (3)$$

where $q_{n,1}$ s are coefficients independent of p_0 and p_1 and

$$a(\sigma,\lambda_l) = \frac{1}{k}\sum_{i=-\infty}^{\infty} \exp\left(-\frac{t_i^2}{2\sigma^2}\right)\cos\frac{2\pi t_i}{\lambda_l} = \exp[-2\pi^2\sigma^2/\lambda_l^2] + O(\Delta t^2) \quad (4)$$

which satisfies $\pm\varepsilon \le a(\sigma,\lambda_l) \le 1$ where ε is a positive machine round-off error. While the second inequality appears to be obvious, the first inequality can only be verified by extensive numerical tests - for modes having $\sigma >> \lambda_l$, the exponential function of Eq. (4) is a positive value close to 0 but the error term, may take a negative small value. In other words, the Gaussian smoothing is approximately diffusive. The remaining high frequency part is

$$y'_j = \sum_{l=0}^{\infty}\left\{(1-a(\sigma,\lambda_l))[c_l \cos\frac{2\pi t_j}{\lambda_l} + d_l \sin\frac{2\pi t_j}{\lambda_l}]\right\} + \sum_{n=0}^{M-2} q_{n,1} t_j^n + O(\Delta t^2) \quad (5)$$

such that the polynomial degree is reduced by 2.

1.2 Iterative Gaussian Smoothing Method

It is easy to show that the transition zone, say the range of λ/σ where $\delta \le a(\sigma,\lambda) \le 1-\delta$, is too wide. In order to shrink this zone, the remaining high frequency part is repeatedly smoothed. Up to m-th cycle the remaining high frequency part will be [10,12-13]

$$y'_{j,m} = \sum_{l=0}^{\infty} b(\sigma,\lambda_l,m)\left[c_l \cos\frac{2\pi t_j}{\lambda_l} + d_l \sin\frac{2\pi t_j}{\lambda_l}\right] + \sum_{n=0}^{M-2m} q_{n,m} t_j^n + O(\Delta t^2) \quad (6)$$

where the second summation will disappear if $2m > M$ and

$$b(\sigma,\lambda_l,m) = (1-a(\sigma,\lambda_l))^m. \quad (7).$$

It can be shown that $0 \le b(\sigma,\lambda_l,m) \le 1 \pm m\varepsilon$, which shows that the iterative procedure is approximately diffusive.

Suppose we applied the iterative Gaussian smoothing method to a data string having two distinct wave lengths, λ_1 and λ_2, where $\lambda_1 < \lambda_2$. If we would like to retain λ_1-wave while filter λ_2-wave , it is intuitive to require that

$$[1-a(\sigma,\lambda_1)]^m \approx \left[1-\exp\left\{-2\pi^2\sigma^2/\lambda_1^2\right\}\right]^m = 1-\delta$$
$$[1-a(\sigma,\lambda_2)]^m \approx \left[1-\exp\left\{-2\pi^2\sigma^2/\lambda_2^2\right\}\right]^m = \delta \quad (8),$$

where the parameter δ can take an arbitrarily small value. The solution of this set of simultaneous equations will give the value of the smoothing factor σ and the number of cycles, m, required to perform the decomposition of the two waves. Note that, as the value of λ_2/λ_1 becomes smaller than 2 (whose corresponding parameters are $\delta = 0.001$, $\sigma/\lambda_1 = 0.7715$ and $m \approx 127$), the required number of the iteration cycle increase exponentially. For example, when $\lambda_2/\lambda_1 = 1.5$, 8134 iteration steps are needed to separate the two waves.

1.3 Filtering Discrete Data String of finite Range

For the problem of filtering a finite data string, (t_i, y_i), $t_i = i\Delta t, i = 0,1,2,...,N$, assume that the polynomial of Eq.(1) is represented by the Fourier expansion. The application of the Gaussian smoothing method will give [13]

$$y'_j = \frac{1}{\bar{k}}\sum_{l=0}^{N-1}\left[c_{l,j,1}\cos\frac{2\pi j}{\lambda_l} + d_{l,j,1}\sin\frac{2\pi j}{\lambda_l}\right] \quad (9),$$

where $\bar{k}_j = \sum_{i=0}^{N-1} e^{-(t_i-t_j)^2/(2\sigma^2)}$ and

$$c_{l,j,1} = c_l - \frac{1}{\bar{k}_j} \sum_{i=-j}^{N-1-j} e^{-t_i^2/(2\sigma^2)} \left[c_l \cos\frac{2\pi t_i}{\lambda_l} + d_l \sin\frac{2\pi t_i}{\lambda_l} \right]$$

$$d_{l,j,1} = d_l - \frac{1}{\bar{k}_j} \sum_{i=-j}^{N-1-j} e^{-t_i^2/(2\sigma^2)} \left[-c_l \sin\frac{2\pi t_i}{\lambda_l} + d_l \cos\frac{2\pi t_i}{\lambda_l} \right]$$

It seems that, since the smoothing is approximately diffusive, there is no information exchange between different Fourier modes after applying the Gaussian smoothing. The corresponding result of applying the iterative Gaussian smoothing to a finite data string is Eq.(9) with $y'_j, c_l, c_{l,j,1}, d_l$, and $d_{l,j,1}$ replaced by $y'_{j,m}$, $c_{l,i+j,m-1}$, $c_{l,j,m}, d_{l,i+j,m-1}$, and $d_{l,j,m}$, respectively.

It can be shown that, for all the interior points where $5\sigma/\Delta t < j < N-1-5\sigma/\Delta t$, all the Gaussian functions at the $i-$th missing point in the range of $-\infty < i < -j$ and $n-j < i < \infty$, respectively, are less than $e^{-12.5} \approx 3.73\times10^{-6}$. Therefore, for these interior points, the smooth responses are almost the same as that estimated by Eq.(3). However, for those points around the two ends where $0 \le j < 5\sigma/\Delta t$ and $N-1-5\sigma/\Delta t < j < N$, the missing points will inevitably make the smooth response deviating from that shown in Eq.(4). For convenience, the location of having deviation being equal to δ is referred to as x_δ. As the iteration cycle increases, x_δ will becomes larger and larger. A careful inspection upon Eqs.(5,6) and (9) reveals that, the error upper bond can be estimated by directly calculating the difference between $y'_{j,m}$s of Eq.(6) and (9), respectively, with unity values of c_l and d_l for $l=1$ because of $a(\sigma,\lambda_1)$ attains the maximum value among all $a(\sigma,\lambda_l)$ s for a given σ. With such an estimation, it is numerically shown that the following formula is a proper estimation in the range of $1 \le m \le 10^4$.

$$x_{0.001}/\sigma \approx 3.4 + 2.83\log_{10}(m) \tag{10}$$

In other words, the error penetration distance increases exponentially with respect to the number of iteration cycle. Extensive numerical experiments show that Eq.(10) applies to both sinusoidal and non-sinusoidal parts.

2. Propose Fast Scheme

2.1 Remove the non-sinusoidal part

Since the non-sinusoidal part of most engineering data can be properly approximated by a polynomial of degree 250, it is reasonable to employ the parameter of λ_2/λ_1=2, say $\delta = 0.001$, $\sigma/\lambda_1 = 0.7715$ and $m \approx 127$. If the acceptable error around the two ends is 0.01, the error penetration distance 5σ is recommended. However, 127 iteration cycles requires a long computing time for many practical problems. In order to speed up both the original and iterative Gaussian smoothing methods, the following procedure is proposed.

1. Properly choose two end points of the data string and connect them with a straight line.
2. Substrate the straight line from the original data string.
3. Find the Fourier spectrum via an FFT algorithm.
4. Determine the transition zone where $\lambda_2/\lambda_1 > 2$ and $\lambda_1 < 0.05T$. Use Eq.(8) to evaluate σ and m.
5. Multiple every Fourier mode by the factor
$$b(\sigma,\lambda_l,m) = \left[1-\exp\left(-2\pi^2\sigma^2/\lambda_l\right)\right]^m \tag{11}$$
6. Evaluate the inverse FFT of the resulting spectrum to obtain the high frequency part.
7. The summation of the straight line and difference between the original data and high frequency part is the smooth part.

If data point number N is larger than 1000, the operation count of the proposed procedure is slightly larger than the sum of forward and inverse FFT because to embed the attenuation factor $b(\sigma,\lambda_l,m)$ and to evaluate σ and m use a count in the same order of $10N$. If the straight line takes the two ends the same as that of the original data, the method is referred to as the well known linear trend removal [14].

2.2 Resulting Error around an End of Proposed Method

If the original data string is infinitely long, the resulting smooth and high frequency parts generated by the iterative Gaussian smoothing and proposed methods without the linear trend removal will have not obvious difference except that caused by the integration error of Eq.(4). It means that the proposed scheme has the same least squares nature as the iterative Gaussian smoothing. The proposed algorithm uses the Fourier spectrum as the basic step to construct the analogy iterative Gaussian smoothing. It means that the Fourier spectrum repeatedly copies the original data to be those data beyond the interested range. However, if the original data point is $N = 2^k - L$, most commercial FFT algorithm adds addition L points with 0 value so as to achieve the fastest efficiency. For the proposed scheme, it is equivalent to repeatedly copy original N points with original data and L points with zero value. The resulting error around the two ends is induced by the difference between the original data (assumed to have infinitely many points) and periodically assumed points. Although this error is somewhat different from the original iterative Gaussian smoothing, numerical experiments show that these modifications do introduce the same error penetration effect as the iterative Gaussian smoothing. In other words, the error penetration distance is increased as the iteration steps m increases. For the parameter arrangement of λ_2/λ_1=2, say $\delta = 0.001$, $\sigma/\lambda_1 = 0.7715$ and $m \approx 127$, the sinusoidal data in the interior region where $5\sigma < t < t_{\max} - 5\sigma$ can be properly approximated by the resulting spectrum.

Principally speaking, the end point data to construct the straight line determine the accuracy of the proposed spectrum generator to certain extent. The reason comes from the fact that, since number of data point is arbitrary, most FFT algorithm feeds zero value to make the total number to be an integer power of 2 or else. In other words, if these ends are just that of the embedded smooth part, the error induced by the finite data length can be effectively suppressed. In this study, an end point is chosen according to the following rule. For example, the left end satisfies the criterion.

$$\begin{aligned} & y_L \le y_{\text{left}} \le y_U \\ & y_L = \bar{y} - k\Delta y_{\max},\ y_U = \bar{y} + k\Delta y_{\max} \\ & \Delta y_{\max} = \text{Max}(y_0, y_1, ...y_A) - \text{Min}(y_0, y_1, ..., y_A) \\ & \bar{y} = [\text{Max}(y_0, y_1, ...y_A) + \text{Min}(y_0, y_1, ..., y_A)]/2 \end{aligned} \tag{12}$$

The region from the left most point and point A should involve at least 2 local maximum and minimum points. The right end is chosen in a similar manner. Then, the data string to be treated by the proposed method is as follows.

$$\begin{aligned} & y(t) = y_{\text{original}} - y_{\text{linear}} \\ & y_{\text{linear}} = y_{\text{left}} + (y_{\text{right}} - y_{\text{left}})(t - t_0)/(t_{\max} - t_0) \end{aligned} \tag{13}$$

2.3 Sharp filter of the remaining high frequency part

Since the remaining high frequency part is generated in the fifth step of the above mentioned procedure, a band-passed limited spectrum can easily being obtained by imposing the desired window. Although this window can be infinitely sharp, there are errors around the two ends. In other words, the corresponding spectrum only accurately reflects the high frequency part in the interior region, where $5\sigma < t < t_{\max} - 5\sigma$.

RESULTS AND DISCUSSIONS

In order to examine the performance of the proposed procedure, the following given data is employed.

$$\begin{aligned} y(t) = & 1.2 + 2t + 0.5t^2 + 0.3e^{-50t^2} \sin(6\pi t) \\ & + 0.4\sin(100\pi t) + 0.2\sin(56\pi t) \\ & + 0.3(1 + 2t + t^2)e^{-0.5t^2} \sin(32\pi t) \end{aligned} \tag{14}$$

where $0 \le t \le 1$. For numerical manipulation, 8190 uniform mesh points were employed to resolute the above function. The original iterative Gaussian smoothing uses the following parameters, $\delta = 0.001, m = 126$, and $\sigma = 0.5$, respectively. The present fast method just employ a simple FFT algorithm with 2^k >8190 data points in which all the artificially embedded points were fed with a 0 value. Those shown in Fig.1 are the original data (thin solid), original smooth part (heavy solid), estimated smooth part via the original iterative Gaussian smoothing (solid) and result of the present approach (dotted line). The corresponding error distributions are shown in Fig.2. In the interior region, both the original and present approaches almost recover the original data. However, around the two ends, both methods introduce error as shown. The proposed method even produces a larger error than the original iterative Gaussian smoothing because of the embedded zero value for points whose index larger than 8190.

Since two ends of the data string of Eq.(14) satisfy the criterion of Eq.(12), the linear trend removal method uses $y(0)$ and $y(1)$ as the left and right ends of the straight line. The resulting smooth part and error distributions are shown in Figs.3 and 4, respectively. A careful inspection upon Figs.1 through 4 reveals that the linear trend removal reduces the magnitude of missing data. Consequently, the magnitude of error source of Fig.3 is smaller than that of Fig.1. Thus the error of Fig.4 in the region around the two ends is obviously smaller than that of Fig.2.

The second test problem employs the following formulas.

$$\begin{aligned} y(t) = & 0.1t - 0.1t^2 + 0.3e^{-50t^2} \sin(6\pi t) \\ & + 0.4\sin(100\pi t) + 0.2\sin(56\pi t) \\ & + 0.3(1 + 2t + t^2)e^{-0.5t^2} \sin(32\pi t) \end{aligned} \tag{15}$$

The magnitudes of the polynomial are approximately 0 so as to inspect the effect of the magnitude of missing data on the error distribution. Figures 5 through 8 show resulting data and error distributions corresponding to those shown in Figs.1 through 4. It is interesting to see that, for the data string of Eq.(15) where magnitudes of the non-sinusoidal and extreme low frequency part around the two ends are slightly scattering from the zero value, the linear trend removal does not have the effect of reducing error. On the other hand, it is obviously that Figs.4 and Figs.8 are rather similar. That means the linear trend removal performs very well in problems whose non-sinusoidal parts are signify-cantly non-periodic. As a consequence, it is recommended to equip the linear trend removal to the proposed fast method.

Figure 9 shows the resulting smooth part estimated by the iterative Gaussian smoothing (heavy solid line) and the proposed method without linear trend removal (dashed line), respectively. The smooth part generated by the iterative Gaussian smoothing has upward going trend around the two end because of the temporary data. On the other hand, the missing data, with zero values, bends the result of present scheme downward as shown. The dashed line shown in Fig.10 uses the present method with linear trend removal which reduces the effect of missing data. Although there is no evidence, the author believes that that estimated by the linear trend removal is more reasonable than others.

The above discussions can be summarized as follows.

1. The proposed fast version has almost the same performance as the iterative Gaussian smoothing performed on the time domain.
2. The proposed method has the same error penetration distance character as the original method. In order to have a small enough error region around the two ends, it is recommended to use parameter set of $\delta = 0.001$, $\sigma/\lambda_1 = 0.7715$ and $m \approx 127$.
3. The operation count of the proposed method is approximately equal to $2N \ln N + kN$, where $k < 100$.

4. The linear trend removal can effectively reduce the error around the two ends.

CONCLUSIONS

The original iterative Gaussian smoothing is approximated by a fast version. The resulting smooth and high frequency parts have the same order of accuracy in the interior region. The error around the two ends of a data string of finite length is similar to that the original method too. The linear trend removal is successfully employed to suppress the error around the two ends. Since the present method has an operation count about twice that of the FFT algorithm, it can be employed as a convenient tool for further studied.

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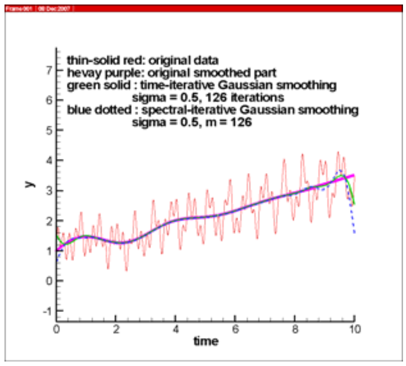

Fig.1 The original data of Eq.(14) and smooth parts estimations: original data is the thin solid line, original smooth part is the heavy solid line, smooth part estimated by the original iterative Gaussian smoothing is the solid line, and that estimated by the present method is the dotted line.

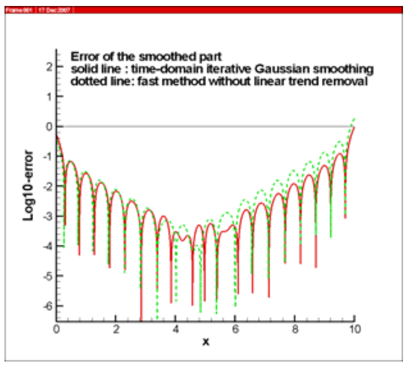

Fig.2 Error distributions of Fig.1: the solid line shows the error estimated by the iterative Gaussian smoothing and the dotted line is the error estimated by the present method.

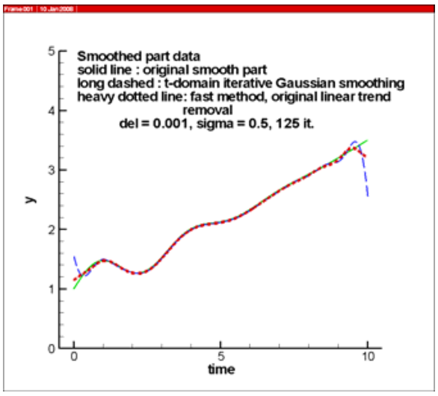

Fig.3 Distributions of smooth parts estimations: original smooth part is the heavy solid line, smooth part estimated by the original iterative Gaussian smoothing is the solid line, and that estimated by the present method is the dotted line.

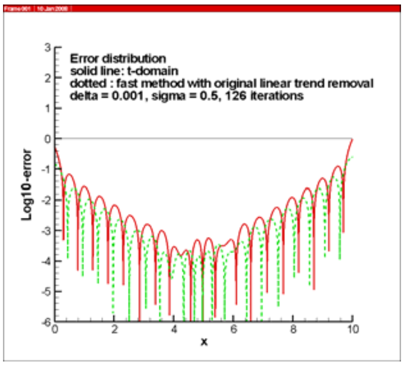

Fig.4 Error distributions of Fig.3: the solid line shows the error estimated by the iterative Gaussian smoothing and the dotted line is the error estimated by the present method.

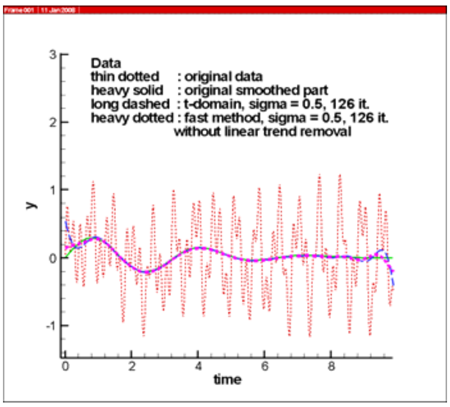

Fig.5 The original data of Eq.(15) and smooth parts estimations: original data is the thin solid line, original smooth part is the heavy solid line, smooth part estimated by the original iterative Gaussian smoothing is the solid line, and that estimated by the present method is the dotted line.

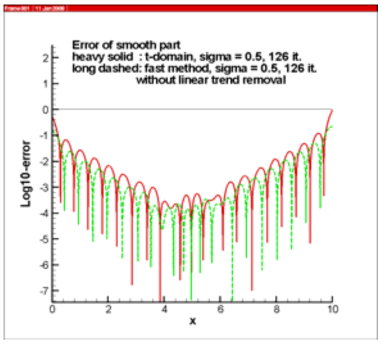

Fig.6 Error distributions of Fig.5: the solid line shows the error estimated by the iterative Gaussian smoothing and the dotted line is the error estimated by the present method.

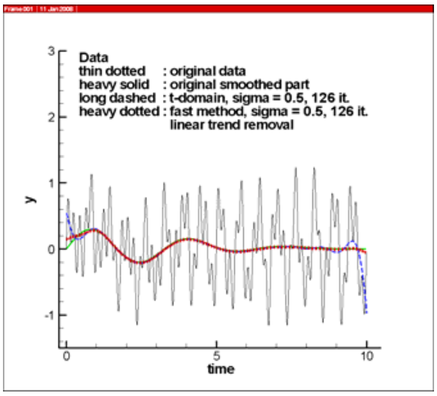

Fig.7 The original data of Eq.(15) and smooth parts estimations: original data is the thin solid line, original smooth part is the heavy solid line, smooth part estimated by the original iterative Gaussian smoothing is the solid line, and that estimated by the present method using the linear trend removal is the dotted line.

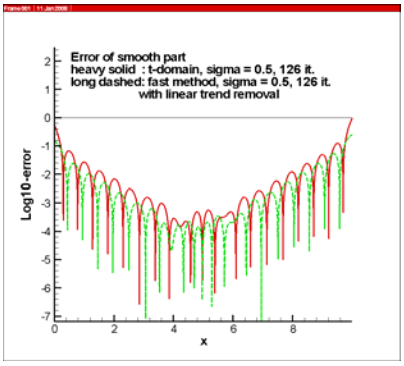

Fig.8 Error distributions of Fig.7: the solid line shows the error estimated by the iterative Gaussian smoothing and the dotted line is the error estimated by the present method.

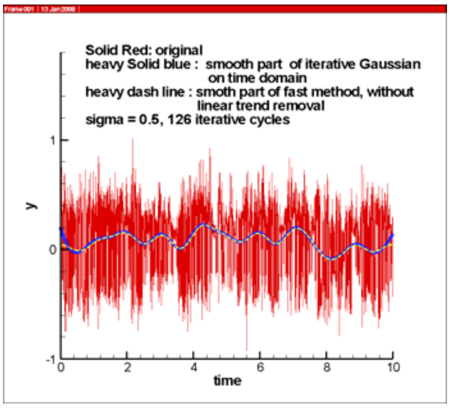

Fig.9 The turbulent data distribution: thin solid is the original data; the heavy line is the smoothed part estimated by the iterative Gaussian smoothing; and the dashed line is estimated by the present method without linear trend removal.

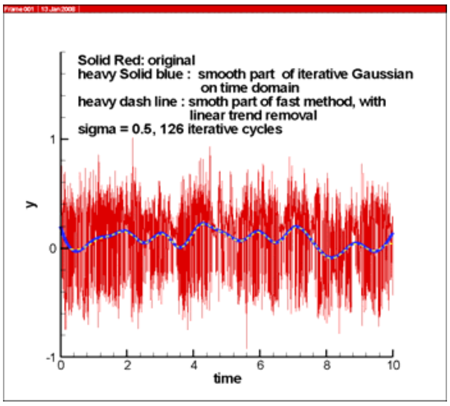

Fig.10 The turbulent data distribution: thin solid is the original data; the heavy line is the smoothed part estimated by the iterative Gaussian smoothing; and the dashed line is estimated by the present method linear trend removal.

使用疊代式高斯平滑法之擴散式快速高低通濾波器

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摘　　要

本文發展使用疊代式高斯平滑法的快速法，其運算次數比離散式快速傳式轉換法的兩倍大一些，比 $2N\ln(N)+100N$ 略小。為了縮小高斯平滑法的濾波轉換窗的窗口寬度，疊代式高斯平滑法最近被發展出來，但其代價是虛需要大量運算時間。為了證明疊代式高斯平滑法有擴散性，之前的文章也推出每個傳式波模的近似衰減因子，並寫出其解析公式。本文直接將該含有疊代次數和平滑參數的衰減因子，乘在每個傳式波模之振幅上，其後作逆快速傳式轉換，發現其內部點的準確度與原疊代式高斯平滑法相當，而數據串兩端的誤差略有不同。本文進一步使用線性移除法，以降低數據兩端的誤差。本文測試數個例子包括一組實驗數據，以說明新方法的可用性。

關鑑詞：疊代式高斯平滑法，頻譜法，線性移除法。